Industrial Agglomeration in City Centers[1]

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I INTRODUCTION

Studies of rank-size are usually based on the relationship between people and the rank of urban hierarchy as pareto distribution that is written generally from the point of view of economic geography. Extensive explanations of the central city theorem in relation to the rank-size have been carried at Parr (1970, 1973), Berry (1961, 1964, 1967) and Tinbergen (1968) and in addition, notes about empirical analysis have been explained by Higgs (1970). Recently Rosen and Penick (1980) have attempted an analysis of city-size distributions for some of the world's countries and Mills, E. S. and B. W. Hamilton (1994) have attempted an estimation of MSA size distributions for sample data of U. S. metropolitan areas. While Beckmann (1958) has constructed a city hierarchies model based on the rural-urban population by using differential equations and, in addition, Beckmann, M. J. and J. McPherson (1970) have applied the model to the rank-size rule. Furthermore, Davis and Swanson (1972) have suggested that differential growth in the labor force of city and rural

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broughts change city-size by use of production function. From the point of view of spatial economics, Lösch (1954) constructed urban spatial structures by use of spatial demand function and his theory is developed using mathematical analysis concretely by Beckmann, M. J. and T. Puu (1990). Almost of literatures about agglomeration and central place theory are reviewed by Mulligan (1984). On the other hand, in studies of agglomeration and spatial economies, Evans (1972) has explained about existence of urban hierarchies by the use of industrial economic theory and Böventer (1975) has suggested an inter city agglomeration model that is in proportion to the population and income of the other city and is in reverse ratio to the distance function to the city such as a gravity model.

Considering rank-size theory and the notion of spatial industrial economics in the above studies, we first use a Cobb-Douglas production function including the level of agglomeration(2) and then derive a rent function for agglomeration in the central city from the profit maximal conditions and the rent in marginal location. Then, estimation of the function based on rank-size is attempted by using the data of population and rent of each central city in England. Finally general estimation of the function is carried out by using data on distance from the airport and rent.

II THE MODEL

First we assume that the value of agglomeration economies to a firm

(2) This is based on Hoover (1948, 1968) and Isard (1956) and recently explained by Richardson (1978), Evans (1985) and Mills and Hamilton (1994).

depends on the distance from the city center and the production function of the firm is assumed to be Cobb-Douglas(3),

$$Q = Q(A(t),\ K,\ L) = A(t)K^{\alpha}L^{1-\alpha}, \tag{1}$$

where $A(t)$ is Level of agglomeration(4), t is the distance from city center, K is capital, L is land, α is coefficient $(0<\alpha<1)$ and constant returns to scale is assumed.

Then the cost constraint of the firm is represented as

$$C = r(t)L + iK. \tag{2}$$

We assume that labor costs are fixed and we abstract the their influence in equation (2).

Where $r(t)$ is rent per unit of site area and i is the rate of investment.

Accordingly the profit of the firm is written as

$$\Pi = pQ - C, \tag{3}$$

where p is product price.

In addition, denoting the capital-land ratio K/L by k and the output-land ratio Q/L by q, equation (1) is rewritten as

$$q = A(t)k^{\alpha}. \tag{4}$$

Next, denoting the profit-land ratio Π/L by π, the profit function can be rewritten as

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(3) See Sveikaukas (1975), Segal (1976) and Moomaw (1981) about other production functions in city.

(4) Strictly speaking, this means agglomeration economies or external effect.

$$\pi = pq(A(t),\ k) - r(t) - ik = pA(t)k^{\alpha} - r(t) - ik \tag{5}$$

Under profit-maximization, the first order conditions are given as

$$\frac{d\pi}{dt} pA'(t)k^{\alpha} - r'(t) = pq\frac{A'(t)}{A(t)} - r'(t) = 0 \tag{6}$$

and

$$\frac{d\pi}{dk} = \alpha pA(t)k^{\alpha-1} - i = \frac{\alpha pq}{k} - i = 0. \tag{7}$$

Dividing equation (6) by equation (7) and rearranging them, we obtain

$$\frac{1}{\alpha}\frac{A'(t)}{A(t)} = \frac{r'(t)}{ik}. \tag{8}$$

Assuming that the firm at marginal distance t_m from the city center, i. e. the urban or city boundary, has a level of agglomeration of one unit, $A(t_m)$, and integrating equation (8) by t, then

$$\frac{1}{\alpha}\int_{u}^{tm}\frac{A'(t)}{A(t)}dt = \frac{1}{ik}\int_{u}^{tm} r'(t)dt, \tag{9}$$

where u is the radius of the city center. Therefore

$$\log A(u) = \frac{\alpha}{ik}(r(u) - r(t_m)). \tag{10}$$

Rearranging equation (10),

$$r(t_m) = r(u) - \frac{ik}{\alpha}\log A(u). \tag{11}$$

If we assume that (i) the number of city center is n and the city center of first rank has the highest level of agglomeration and the highest rent and (ii) the rent in each marginal distance of all industrial cities is the same, $r_1(t_m)=r_2(t_m)=\cdots=r_n(t_m)$[5], then,

$$\begin{aligned} r_1(u)-\frac{ik}{\alpha}\log A_1(u) &= r_2(u)-\frac{ik}{\alpha}\log A_2(u) \\ &\ \ \vdots \\ &= r_n(u)-\frac{ik}{\alpha}\log A_n(u). \end{aligned} \tag{12}$$

Arranging equation (12) in general form, we have

$$r_1(u)-r_n(u)=\frac{ik}{\alpha}(\log A_1(u)-\log A_n(u)) \tag{13}$$

or

$$r_n(u)=r_1(u)-\frac{ik}{\alpha}\log\frac{A_1(u)}{A_n(u)}. \tag{14}$$

Finally, if $A(u)$ can be measured, we can estimate equation (14) by use of maximum likelihood technique and the estimated parameter yield, $\frac{ik}{\alpha}$ or the growth rate of production to the marginal investment of capital (inverse of $\frac{ik}{\alpha}$[6]) is obtained.

Summarizing the model, from equation (14), first an increase in the

(5) It should be noted that boundaries between industrial urban areas are not necessarily tangential.

(6) This means the inverse of $\frac{ik}{\alpha}=\frac{ik\frac{\Delta k}{k}}{\frac{\Delta q}{q}}=\frac{i\Delta k}{\frac{\Delta q}{q}}$

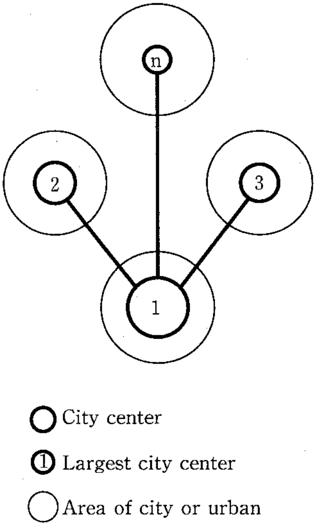

Figure 1 Location Map in The model

level of agglomeration in the city center of first rank decreases the rent in the city center of the nth rank and, conversely, a decrease in the level of agglomeration in the city center of first rank increases the rent in the city center of the nth rank. However, a relative decrease in the level of agglomeration in the city center of the nth rank decreases the rent of the city center and conversely, a relative increase of level of agglomeration in the city center of the nth rank increases the rent of the city center. Second the model suggests that if $\frac{ik}{\alpha}$ is relatively high, the group[7] tends to constitute the city centers which need more marginal investment per site area and if $\frac{ik}{\alpha}$ is relatively low, the group tends to be made up of city

(7) This denotes objective area of analysis such as metropolitan area or country.

centers which need a larger site area. Third, as shown in Figure1, it is an important point that the model is not regulated by geographical condition of sequential intra-urban boundaries but by the rent of the boundary in a city or an urban area.

III APPLIED ANALYSIS IN CENTRAL CITIES OF ENGLAND

1 Rent Model based on Rank-Size Rule

In this section, first assuming that level of agglomeration in the city center is in proportion to its population, equation (14) is rewritten as

$$r_n(u) = r_1(u) - \frac{ik}{\alpha}\log\frac{P_1(u)}{P_n(u)}, \qquad (15)$$

where $P_1(u)$ and $P_n(u)$ denote the population in each city center of first rank and the nth rank.

The hypothesis is thus a decrease in city center-rank based on the scale of population, decreases rent in order from the first rank to the nth rank. Equation (15) can be estimated by the maximum likelihood method.

We consider groups of central cities in relation to rank-size from the rent data in central cities of England(8) by considering the city center as the central city because it is difficult for us to find data of rent and population of the city center.

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(8) Here industrial rent data of 50 centres by Jones Lang Wootton May 1993 is used but because cities in Greater London and in surroundings of that have about same scale of population and rent, we excepted for central cities within 50km from Hounslow airport according to the implicit assumptions that rent and population as agglomeration must be higher than any other cities in industrial urban area.

Table1 Central Cities in Rank Order by Group

groupA :	1. Birmingham, 2. Leeds, 3. Sheffield, 4. Liverpool
groupB :	1. Tonbridge, 2. Southampton, 3. Solihull, 4. Northampton
groupC :	1. Basingstoke, 2. Brighton, 3. Maidstone, 4. Norwich

Note : It is noted that Solihull and Norwich are relatively far from each central city of first rank.

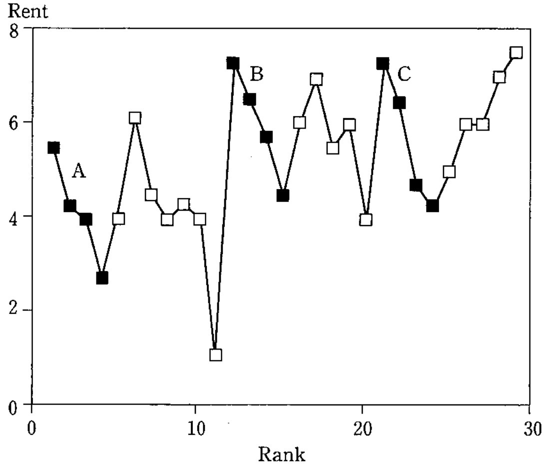


Figure 2 Rent and Rank Size

Note : The date of population in each city to obtain rank size is based on 1991 Census Preliminary Report for England and Wales and the unit of date of rent is £/ sq. ft. p. a. .

Seeing about central cities of black square of right downward steep from Figure2, the following groups are classified[9] as shown in Table1.

From Figure2 and Table1, the central city of first rank in groupA is

(9) As shown in Figure 2, it should be noted that the groups that have two or three central cities are not adopted from the point of view of considering geographical characteristics of industrial area.

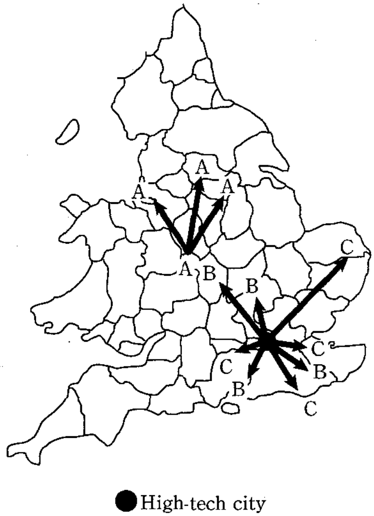
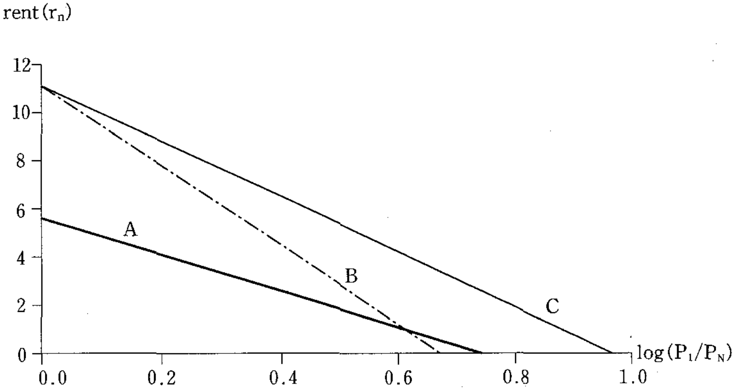


Figure 3 City Map by Hierarchies based on The Model



A group : $r_n = 5.5 + 7.491 * \log(P_1/P_n)$ with $R^2 = 0.661$

B group : $r_n = 11 + 16.388 * \log(P_1/P_n)$ with $R^2 = 0.479$

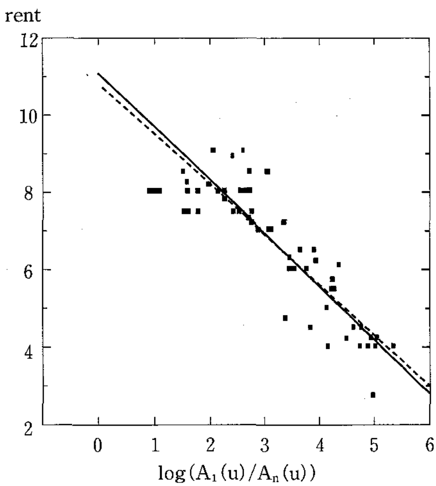
C group : $r_n = 11 + 11.502 * \log(P_1/P_n)$ with $R^2 = 0.295$

Figure 4 Rent Equations-Maximum Likelihood Estimates

Birmingham that saves some cities of former metropolitan counties. On the other hand, from Figure2 and Figure3, each central city of groupB and C is obviously located on a circumference of Heathrow airport, which is regarded as the city center of each group in the analysis below, the central city denotes a unique city which is unified by the high-tech towns of Hounslow, Bracknell and Slough and here after we call the central city, that is the city of first rank, "the High-tech city". Then from this classification, it is considered that *competition of land prices has a close relationship to constitutional characteristic of industrial area.* Next from Figure4, the coefficient of groupB $\frac{ik}{\alpha}$ as absolute value is higher in absolute terms than each tendency of groupA and C. This implies that *the central cities in groupB have an industrial structure which needs larger investment per site area and on the other hand the central cities in each of groupA and C have an industrial structure which needs larger site area such as manufacturing industry.* However, it should be noted that $\frac{ik}{\alpha}$ is estimated from very few sample observations and depends on the scale of population of the central city of first rank.

2 Rent Model without being based on Rank-Size Rule

In this section, we first assume that the city center of first rank is Hounslow, whose center is Hearthrow airport and then the level of agglomeration in each central city is measured by inverse of line distance from Heathrow airport because the transportation costs of access to the airport and the big market of London is reduced. Finally, equation (14) is estimated by using both the maximum likelihood method and the least squares method and the estimated result for the former is :



Line : Rent function by maxmum likelihood method
Dot line : Rent function by least squared method

Figure 5 Rent Function-Maxmum Likelihood Method, Least Squared Method Estimates

$$r_n = 11 - 1.371 * \log(A_1(u)/A_n(u)) \text{ with } R^2 = 0.781 \quad n = 49$$

For the latter method, with corrected t statistics in brackets, we obtain :

$$r_n = \underset{(31.721)}{10.768} - \underset{(-13.151)}{1.307} * \log(A_1(u)/A_n(u)) \textit{ with } R^2 = 0.783 \quad n = 50$$

Summarizing these estimated results, each estimated function has similar coefficient and the goodness of fit is high as shown in Figure5. Therefore, considering the level of agglomeration in this model as the distance from the airport is very significant and from each estimated value of ik/α, production growth rate to investment per site area α/ik is probably between 70 percent and 80 percent as an average of each central city in England.

IV CONCLUDING REMARKS

We first constructed a production function including agglomeration and then derived a function for rent-agglomeration in relation to rank-size from firm's profit-maximum conditions and the condition that the rent in the marginal location of each industrial urban area is same. In addition we reconstructed the function to estimate by replacing agglomeration by population. Next we found groups of central cities that have a reasonable fitness for the model from data on rent and population of each city in England and applied the function to these groups, although the sample sizes are very small. The group is included in former metropolitan counties and the two groups are included in surrounding areas of high-tech cities of Greater London and the central city of first rank size in the first step was Birmingham. The central city in the latter was defined as High-tech city unified by Hounslow, Bracknell and Slough. We estimated the function under these settlements and found that there is a group of central cities which have larger investment per site area and there are two groups of central cities which have larger site area but it is difficult for us to conclude whether such results are due to accident or not. On the other hand, the rent function without being based on rank-size is estimated by the use of two regression analysis methods and by comparering these estimated functions, we found that the goodness of fitness for these function is high and the production growth rate for investment per site area is approximately between 70 percent and 80 percent. It should be noted, however, that *rent in each marginal location (or boundary) of* *sequential central cities which have high level of agglomeration is not necessarily equal to that of sequential central cities which have lower level of agglomeration and the scale of the parameter is dependent on the size of sample and the index of agglomeration.* Accordingly in the future, we must attempt empirical analysis for our model using more sample data on industrial rents and time series data on each central city, by considering geographical location of the central city. Finally from the point of view of the level of agglomeration in industrial central city, the level of transportation networks, institution and service as the level of agglomeration should be discussed.

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