

Optimum Level of Development in a Residential Area¹

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We first derive a function to estimate the external effects to the household from maximizing the household's utility subject to a budget constraint. Then, the derived function is applied to residential data for the biggest metropolitan area in Japan. Finally applying cost-effectiveness analysis to the developer of public utilities, an optimum development cost function is simulated. We find that the developer must increase the optimum development cost to reach the fifth level, but from this level, can decrease cost gradually. This implies that the optimum cost for renewal in the central city is higher than for development in suburbs. In addition, the developer must have higher development costs in order to develop the residential area with higher externality, i.e., higher development effects.

I. INTRODUCTION

The traditional framework for the cost-benefit method in the urban area is due to Rothenberg [1965, 1967] who carried out an ex post evaluation of three urban renewal projects in Chicago. As representative methods to evaluate the benefit in residential area, benefit can be

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measured as the change in consumer's surplus using a Marshallian demand curve or alternating by tenant benefit using the Hicksian price equivalent. The former is studied by Sumka & Stegman [1978] and Kraft & Kraft [1979], while the latter has been studied by DeSalvo [1971, 1975]. He measured benefit from an indirect utility function derived from a Cobb-Douglas utility function and applied the method for a New York city. In addition, Murray [1975] and Olsen & Barton [1983] estimated benefit from sample data on public housing tenants by applying DeSalvo's model. Flowerdew & Rodriguez [1978] used contingent valuation as an alternative method of the Hicksian measure to estimate the benefit from replacing a part of Victorian residential properties with council housing. Recently, the way to measure benefit in residential areas is considered by Hammond [1987] and Schofield [1987].

From the above studies, we first construct a residential location model based on Alonso [1964] and Muth [1969], using the household's utility function including the level of externalities as Hicks neutral. Then from the utility maximum conditions, a function for household externalities is derived and the derived function is applied to residential data for the biggest metropolitan area in Japan. Finally we develop optimum cost function for cost-effectiveness analysis for developers of public utilities, which is simulated.

II. THE MODEL

The residential location model is based on the following assumptions:

- (1) Household utility function is composed of an external effect²,

² This indicates level of external effect concerning urbanization economies such as transportation service or exist of city park.

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composite goods³ and site area. The utility function is exponential in form and homogeneous of degree one.

(2) The household budget is composed of expenditure on the composite goods and rent.

Under these assumptions, the household's utility function is represented as

$$u = A(d)z^\alpha q^\beta, (\alpha + \beta = 1 \text{ from assumption (1)}) \quad (1)$$

where $A(d)$ is the externality, d is level of development, z is quantity of the composite goods, q is site area, α and β are the elasticities of z and q respectively.

The budget constraint is given as

$$y = p_z z + r(d)q, \quad (2)$$

where y is income, p_z is price of the composite goods, $r(d)$ is rent of the apartment per site area.

Then in order to maximize household's utility, we form the Lagrangian expression

$$L = A(d)z^\alpha q^\beta + \lambda(y - p_z z - r(d)q). \quad (3)$$

First order conditions are as follows:

$$\frac{\partial L}{\partial d} = \frac{A'(d)}{A(d)} u - \lambda r'(d)q = 0, \quad (4)$$

$$\frac{\partial L}{\partial z} = \frac{\alpha}{z} u - \lambda p_z = 0, \quad (5)$$

$$\frac{\partial L}{\partial q} = \frac{\beta}{q} u - \lambda r(d) = 0, \quad (6)$$

³ Composite goods here are composed of every consumption goods except for "site area".

$$\frac{\partial L}{\partial \lambda} = y - p_z z - r(d)q = 0. \quad (7)$$

From equation (5) and (6),

$$(\alpha + \beta)u = u = \lambda(p_z z + r(d)q). \quad (8)$$

In addition, substituting equation (7) into equation (8)

$$u = \lambda y. \quad (9)$$

Then dividing equation (4) by equation (9),

$$\frac{A'(d)}{A(d)} = \frac{r'(d)q}{y}. \quad (10)$$

Integrating equation (10) by d ,

$$\log A(d) = \frac{r(1)q}{y} \left(\frac{r(d)}{r(1)} - 1 \right)$$

or

$$A(d) = \exp \left(\frac{r(1)q}{y} \left(\frac{r(d)}{r(1)} - 1 \right) \right), \quad (11)$$

where development at the first level $A(1)$ is expressed as one unit. Furthermore, From equation (11), if $0 < \frac{r'(d)}{r(1)}$ and $\frac{r''(d)}{r(1)} < 0$, then $0 < A'(d)$ and $A''(d) < 0$. Here assuming that the development cost per household is only determined by the level of development, whose function is linear, the function is written as

$$C(d) = C(1) + \Delta(d - 1), \quad (12)$$

where $C(1)$ is the cost of development at the first level per household and Δ is the development cost per level of development. Then calibrating equation (12) by reconstructing the projected data as $C(1) = 1$ one unit cost per household, we have

$$c(d) = 1 + \delta(d - 1). \quad (13)$$

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And dividing equation (13) by $c(1)$ ($= 1$) to unify each scale of $c(d)$ and $A(d)$,

$$\frac{c(d)}{c(1)} = \frac{1 + \delta(d-1)}{c(1)} = \bar{c}(d) = 1 + \bar{\delta}(d-1). \quad (14)$$

Consequently, $A(d)$ and $\bar{c}(d)$ can be shown by Figure 1.

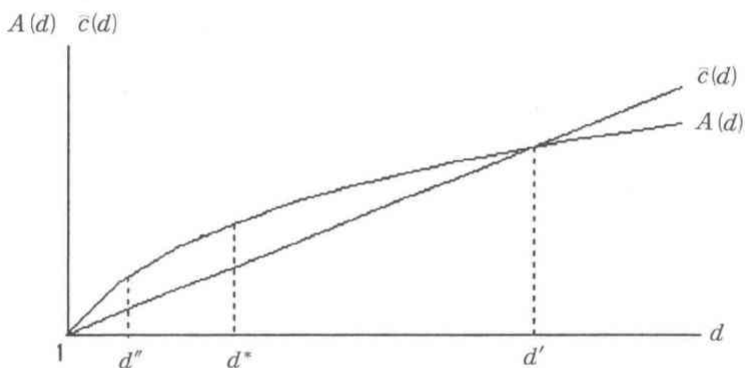


Figure 1 Effectiveness and Cost

From the point of view of cost-effectiveness analysis, the optimum level of development is determined at the point where $A'(d) = \bar{c}'(d)$, that is, d^* . In the case without constraints on cost and urban planning, the level is d' and in the case of each strict constraints, the development may be d'' .

Next, from equation (11) and equation (14), the optimum development condition is $A'(d) = \bar{c}'(d)$; accordingly the optimum level of development is derived by solving

$$\left(\frac{r(1)q}{y} \left(\frac{r(d)}{r(1)} - 1\right)\right) \left(\frac{r(1)q}{y} \frac{r'(d)}{r(1)}\right) \exp\left(\frac{r(1)q}{y} \left(\frac{r(d)}{r(1)} - 1\right)\right) = \bar{\delta}. \quad (15)$$

Here after we call equation (15) 'the optimum development cost function' and $\bar{\delta}$ 'the optimum development cost'. However, it should be noted that $\bar{\delta}$ denotes value without measure.

III. SIMULATION ANALYSIS

In this section, we attempt simulation analysis for the optimum development cost function.

First estimating $r(d)/r(1)$ from data⁴ on residential location of Tokyo to which is in the biggest metropolitan area in Japan, by substituting distance to the CBD for d under the assumption that the distance to CBD is in proportion to level of urbanization development,

$$\frac{r(d)}{r(1)} = d^{0.2}. \quad (16)$$

In estimate the correlation coefficient is 0.762, the t value is 8.066 and the number of cities is 49.

Substituting equation (16) and which is average value in Japan for equation (15),

$$0.018d^{-0.8}(d^{0.2} - 1)\exp(0.3(d^{0.2} - 1)) = \bar{\delta}. \quad (17)$$

Equation (17) is shown in Figure 2. The optimum development cost increases steeply up to the fifth level, but from that level, the cost decreases slowly as the level rises. This suggests that the developer must increase optimum development cost to attain development at the fifth level in cost-effectiveness analysis, but from that level, can decrease it gradually.

Then because generally $\frac{r(1)}{y}$ in the central city is higher than in the suburbs, assuming that $A(d)$ is constant, $\frac{r(1)}{y} = 0.4$ in the central city

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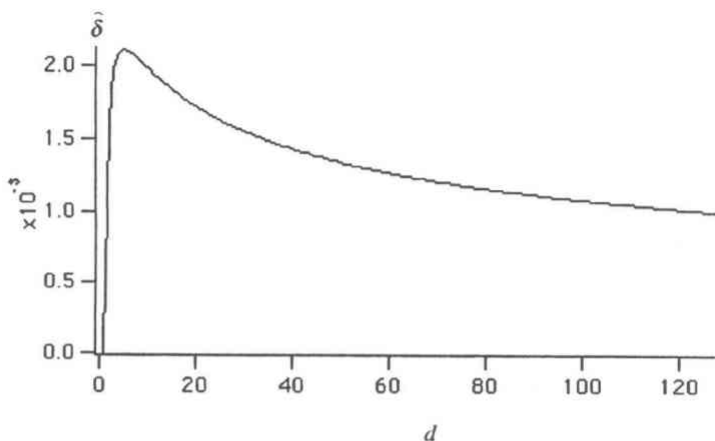
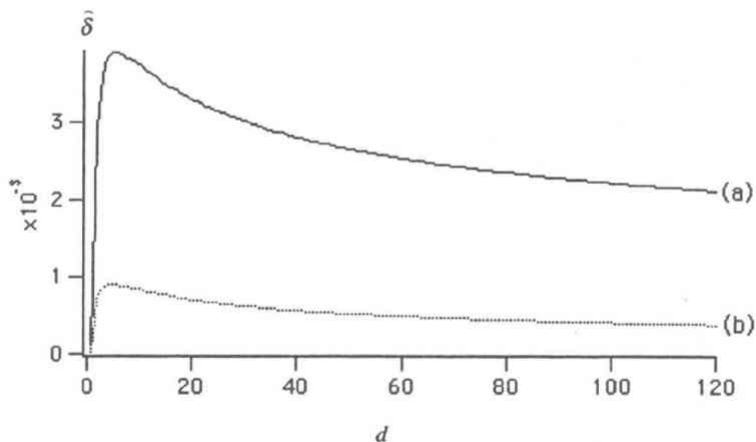


Figure 2 Optimum Development Cost Function



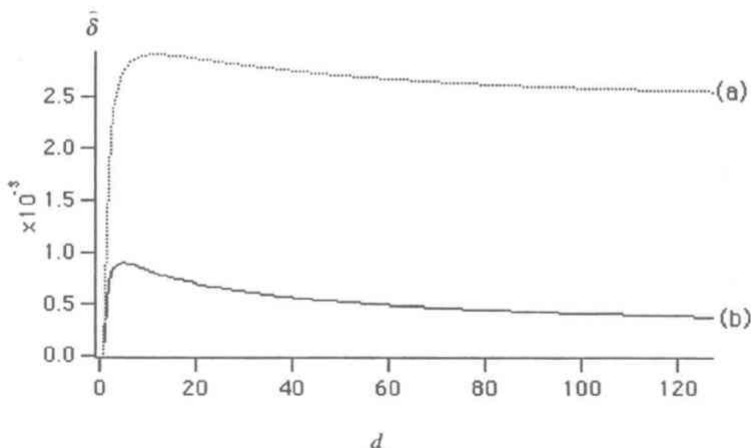
(a) $\hat{\delta} = 0.032 \cdot d^{-0.8} \cdot (d^{0.2} - 1) \cdot \exp(0.4 \cdot (d^{0.2} - 1))$ in case of $\frac{r(1)}{y} = 0.4$

(b) $\hat{\delta} = 0.008 \cdot d^{-0.8} \cdot (d^{0.2} - 1) \cdot \exp(0.2 \cdot (d^{0.2} - 1))$ in case of $\frac{r(1)}{y} = 0.2$

Figure 3 Optimum Development Cost Function by $\frac{r(1)}{y}$

and $\frac{r(1)}{y} = 0.2$ in suburb, the optimum development cost function in the central city is higher than in suburbs as shown by Figure 3. This suggests that the optimum cost for renewal in the central city is higher than for development in the suburbs if the renewal and the development are executed by the same plan and same procedure at the same time.

Finally, assuming that $\frac{r(1)}{y}$ is constant and considering residential areas which have different elasticities of utility with respect to the level of development, i.e., $A(d) = d^{0.2}$ and $A(d) = d^{0.3}$, the optimum development cost function in a residential area with $A(d) = d^{0.3}$ is higher than that in a residential area with $A(d) = d^{0.2}$ as shown by Figure 4. This suggests that the developer must have higher development costs in order to develop residential areas with greater external



(a) $\hat{\delta} = 0.012 \cdot d^{-0.7} \cdot (d^{0.3} - 1) \cdot \exp(0.2 \cdot (d^{0.3} - 1))$ in case of $A(d) = d^{0.3}$

(b) $\hat{\delta} = 0.008 \cdot d^{-0.8} \cdot (d^{0.2} - 1) \cdot \exp(0.2 \cdot (d^{0.2} - 1))$ in case of $A(d) = d^{0.2}$

Figure 4 Optimum Development Cost Function by $A(d)$

effects, i.e., greater development effects if the developments are executed by the same plan and same procedure at the same time.

IV. CONCLUDING REMARKS

Taking account of the residential location, we first specified an exponential utility function to include Hicks neutral externalities effect. Second, we derived a function to estimate level of household externalities from constraint utility maximization. Third, the derived function was applied to cost-effectiveness analysis for developer of public utilities. Finally, using residential data on the metropolitan area in Japan and assuming a linear development cost function, the optimum development cost function was simulated. We found that, first, the developer must increase optimum development cost up to the fifth level, but from that level, can decrease cost gradually. Second the optimum cost for renewal in the central city is higher than that for development in the suburbs. Third the developer must have higher development costs in order to develop a residential area with higher external effects, i.e., higher development effect. Furthermore, it is possible to make a plan for the development of a new town or transportation system by this model and to analyze optimum timing of development by replacing the level of development by the timing of development. However it should be noted that the solution of the optimum does not exist in the case where relative cost function is greater or equal to the relative effectiveness function. In future work, in order to apply the model not only to public utility developers but also private developers, we need to construct cost-effectiveness model through general equilibrium analysis and from a non-linear development cost function.

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